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LETTER TO THE EDITOR

Quasiparticle damping in two-dimensional tight-binding Fermi systems

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Abstract. The quasiparticle damping and the phase space are studied through the Fermi liquid analysis for a non-half-filled two-dimensional weakly interacting tight-binding Fermi system. The $T^2 \ln T$ or $\omega^2 \ln \omega$ dependence of the damping is obtained, in the same way as for 2D Fermi systems with effective-mass dispersions.

In recent years there has been a great deal of interest in the question of whether or not Landau's mean field theory of Fermi liquids is valid in two-dimensional (2D) Fermi systems [1], especially since the discovery of high-temperature superconductors. A Fermi surface [2] and anomalies in the normal state of Cu–O high-temperature superconductors [3] have been observed, the latter being unlike those observed in any other metals and not as expected for a Fermi liquid. So there is a suggestion that the normal state of the high-temperature superconductors may not be an ordinary Landau–Fermi liquid [4–6]. Two of the basic factors that make the systems non-Fermi-liquid-like may be the low dimensionality and the tight-binding energy structure of the systems. It thus seemed important to us to study the 2D tight-binding Fermi systems to seek a clue to the question.

Low dimensionality may lead to singular consequences. The three-dimensional (3D) interacting Fermi systems are well described by the Fermi liquid theory [7]. The quasiparticle damping, namely the imaginary part of the single-particle self-energy $\mathrm{Im}\,\Sigma$ is proportional to T^2 or ω^2, where T is the absolute temperature and ω is the energy measured from the Fermi energy. The analysis [8] of the one-dimensional (1D) electron systems by the use of the Fermi liquid theory gives $\mathrm{Im}\,\Sigma \sim T$ or ω, which implies the disappearance of the discontinuity of the momentum distribution n_k and the quasiparticle weight $z_{k_F} = 0$. So the 1D interacting Fermi systems are non-Fermi-liquid-like. The ground state of 2D interacting Fermi systems is not completely clear at the present time. The calculations [9–13] for 2D fermions with an effective-mass dispersion model show that the leading term of $\mathrm{Im}\,\Sigma$ is proportional to $T^2 \ln T$ or $\omega^2 \ln \omega$. In addition, we can show that the quasiparticle weight $z_k \neq 0$, and n_k has a discontinuity at $k = k_F$. Thus, although the 2D quasiparticle damping is stronger than that in 3D systems, it is still small compared with the quasiparticle energy given by T

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or ω, i.e., the quasiparticle lifetime near the Fermi energy is long enough. In other words, the quasiparticles are well defined and Landau–Fermi liquid theory is valid in such 2D systems. The same conclusion is obtained for 2D itinerant Fermi systems [12]. In this letter we will study the quasiparticle damping and the phase space for the non-half-filled 2D weakly interacting tight-binding Fermi systems. The $T^2 \ln T$ or $\omega^2 \ln \omega$ dependence of the damping is obtained, as it is for 2D Fermi systems with effective-mass dispersions.

Tight-binding energy structure may also lead to singular consequences. By including the electron–electron scattering terms corresponding to a typical nesting momentum for the 2D tight-binding model, the quasiparticle damping is shown [6, 14] to be proportional to T or ω, which is different from what is found for both the 3D Landau–Fermi liquid and the 2D Fermi liquid with effective-mass models. This is true for perfect-nesting Fermi systems. Here we consider the non-half-filled case with the following Hamiltonian:

$$\hat{H} = \sum_{k\sigma} E_{k\sigma} \hat{C}^{+}_{k\sigma} \hat{C}_{k\sigma} + U \sum_{k,k';q\neq 0} \hat{C}^{+}_{k'+q\uparrow} \hat{C}^{+}_{k-q\downarrow} \hat{C}_{k\downarrow} \hat{C}_{k'\uparrow} \tag{1}$$

where $U > 0$ denotes the on-site Coulomb repulsion and $\hat{C}^{+}_{k\sigma}(\hat{C}_{k\sigma})$ is the creation (annihilation) operator of an electron with momentum k and spin σ within a tight-binding band

$$E_k = -2t\left[\cos(k_x) + \cos(k_y)\right] \tag{2}$$

with $8t$ the width of the band. The quasiparticle damping, i.e., the imaginary part of the single-particle self-energy, is given by [15]

$$\begin{aligned} \mathrm{Im}\Sigma(k,\omega) = &\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^1}{2\pi} \left(\coth \frac{\omega-\omega'}{2T} - \tanh \frac{\omega'}{2T}\right) \\ &\times \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega''}{2\pi} \left(\tanh \frac{\omega''}{2T} - \tanh \frac{\omega''+\omega-\omega'}{2T}\right) \sum_{k',q} \mathrm{Im}\ G(k-q,\omega') \\ &\times \mathrm{Im}\ G(k',\omega'')\mathrm{Im}\ G(k'+q,\omega''+\omega-\omega')\Gamma(k,k';k'+q,k-q)|^2 \end{aligned} \tag{3}$$

where $G(k,\omega)$ denotes the Green's function and $\Gamma(k,k';k'+q,k-q)$ denotes the vertex part. The conventional T^2-term is given by [16]

$$\mathrm{Im}\ \Sigma^{(2)}(k,0) = \tfrac{1}{2}(\pi T U)^2 \sum_q \mathrm{Im}\ G(k-q,0)\eta(q) \tag{4}$$

where $\eta(q)$ is the phase space available for scattering quasiparticles through wavevector q from k to $k+q$,

$$\eta(q) = \frac{1}{\pi^2} \sum_k \mathrm{Im}\ G(k,0)\mathrm{Im}\ G(k+q,0). \tag{5}$$

We consider the second-order term in U, namely within the Born approximation, where Green's function G and the vertex part Γ are replaced by the corresponding unperturbed values, respectively. In this limit, from (2) and (5) one finds

$$\eta(q) = \left[1/(2t)^2\right]\xi(q,f) \tag{6}$$

with

$$\xi(\boldsymbol{q},f) = 1/\{\sqrt{2}\pi^2[(1-\cos(q_x+q_y)-2g^2(q_x,q_y;f))^{1/2} + (1-\cos(q_x-q_y)-2g^2(q_x,-q_y;f))^{1/2}]\} + (q_x \leftrightarrow -q_x) \qquad (7)$$

and

$$g(q_x,q_y;f) = \tfrac{1}{2}(\sin(q_x)+\sin(q_y)) - f^2/[\cot(q_x/2)+\cot(q_y/2)] + \left[\left\{f^2/[\cot(q_x/2)+\cot(q_y/2)]\right\}^2 + \left[f\sin(q_x-q_y)/2\right]^2\right]^{1/2}. \qquad (8)$$

In (8) $f = -\mu/2t$, with μ the chemical potential. Obviously this phase space is strongly dependent on the wavevector q and the chemical potential μ. For the half-filled-band case, $\eta(q) \sim (\cos(q_x)-\cos(q_y))^{-1}$, which is divergent along the $[11\bar{1}]$ or [110] direction. For the nearly half-filled cases, there is ridge of $\eta(q)$ along the entire $[11(\bar{1})0]$ direction arising from transitions along the edge of the Fermi surface. These same transitions will contribute strongly to the imaginary part of the susceptibility Im $\chi(\boldsymbol{q},\omega)$ for small ω. We think that such a strong q dependence of Im $\chi(\boldsymbol{q},\omega)$ must be taken into account in modelling the recent nested-Fermi-liquid model [6]. For nearly empty or nearly filled cases, $\eta(q) \sim (q\sqrt{4k_F^2-q^2})^{-1}$, which is divergent for $q = \sqrt{q_x^2+q_y^2} = 0$ or $2k_F$.

We next turn to a calculation of the quasiparticle damping. We now need to include the quasiparticle scattering terms corresponding to all the possible transfer momenta as illustrated above. Using (6)–(8), within the Born approximatioin we find from (4) that

$$\mathrm{Im}\ \Sigma^{(2)}(\boldsymbol{k},0) \to \infty. \qquad (9)$$

It is easy to see that the divergence is due to the special scattering processes. For example, for nearly half-filled cases, the transfer of momentum is given as $q_x = q_y$, while for nearly empty or nearly filled cases, the transfer of momentum is 0 or $2k_F$. This divergence implies that the leading term of the imaginary part of the self-energy is not proportional to T^2 or ω^2.

On the other hand, we can show that the leading term of the imaginary part of the self-energy is not proportional to T or ω. This result is independent of the details of the dispersion, so long as the Green function has the quasiparticle form, i.e. Im $G(\boldsymbol{k},\omega) \sim z_{\boldsymbol{k}}\delta(E^*_{\boldsymbol{k}}-\mu-\omega)$, where $E^*_{\boldsymbol{k}}$ is the renormalized quasiparticle energy. At $T=0$, we expand Im $\Sigma(\boldsymbol{k},\omega)$ to the T- or ω-order. For $f \neq 0$, the corresponding coefficient is given by

$$[\partial\ \mathrm{Im}\ \Sigma(\boldsymbol{k},\omega)/\partial\omega]|_{\omega=T=0} = [\partial\ \mathrm{Im}\ \Sigma(\boldsymbol{k},\omega)/\partial T]|_{T=\omega=0} = 0. \qquad (10)$$

Thus, we may write

$$\mathrm{Im}\ \Sigma(\boldsymbol{k},\omega) \sim \begin{cases} -[\pi T U^2/(2t)^2]F(\boldsymbol{k},\pi T/2t) & \text{if } T \gg |\omega| \\ -[\omega U^2/(2t)^2]F(\boldsymbol{k},\omega/2t) & \text{if } T \gg |\omega| \end{cases} \qquad (11)$$

where $F(k,\epsilon)$ is vanishingly small but $F(\epsilon)/\epsilon$ is divergent in the $\epsilon \to 0$ limit, as indicated by (10) and (9). The leading term of $F(k,\epsilon)$ is given by

$$F(k,\epsilon) = \frac{\pi\epsilon}{2}\sum_q \xi(k+q,f)\delta(f-\epsilon-\cos(q_x)-\cos(q_y)). \qquad (12)$$

In some limiting cases, using (7) and (8) in (12), we get the following results for the small-ϵ limit. For example, for nearly empty and nearly filled cases,

$$F(k,\epsilon) \simeq [3\epsilon/32\pi^3(2-f)]\ln|(2-f)/\epsilon|. \qquad (13)$$

For nearly quarter-filled cases,

$$F(k,\epsilon) \simeq (\epsilon/4\pi^3)\ln|1/\epsilon|. \qquad (14)$$

For nearly half-filled cases,

$$F(k,\epsilon) \simeq (3\sqrt{3\epsilon}/32\pi^3\sqrt{f})\ln|\sqrt{f}/\epsilon|. \qquad (15)$$

The numerical calculations show that $F(k,\epsilon)$ is proportional to $\epsilon \ln \epsilon$ for the general cases ($f \neq 0$). Clearly, such a quasiparticle damping (equations (11)–(15)) is small compared with the quasiparticle energy given by T or ω. Therefore we can define quasiparticles in the non-half-filled 2D weakly interacting tight-binding Fermi systems. This is the same as for the 2D Fermi systems with effective-mass dispersions where the Fermi liquid theory is valid [9–13].

We now have to discuss the effect of including the higher-order terms in U. As shown in (3), the effect on Im Σ results from both the renormalization for the Green's function by z_k and E_k^* and the vertex function $\Gamma(k,k';k'+q,k-q)$. We note that the quasiparticle weight z_k is not included in the delta function in the imaginary part of the Green's function. So the only effect on Im Σ results from the renormalized quasiparticle energy E_k^*. We know that the $T^2 \ln T$ or $\omega^2 \ln \omega$ dependence of the damping is not altered by the renormalization for the quasiparticle energy E_k^*, and thus by the renormalization for the Green's function. As for the vertex function $\Gamma(k,k';k'+q,k-q)$, since it is difficult to determine its detailed form in 2D tight-binding systems, its effect on Im Σ is currently unknown. A further study is needed for the vertex function Γ. It should be pointed out that the possible singularities in the vertex function may also make the system non-Fermi-liquid-like.

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References

[1] Anderson P W 1990 *Phys. Rev. Lett.* **64** 1839
[2] Imer J-M, Patthey F, Bardel B, Schneider W-D, Baer Y, Petroff Y and Zettl A 1989 *Phys. Rev. Lett.* **62** 336
[3] See, e.g.,
Kamimura H and Oshiyama A 1989 *Mechanisms of High Temperature Superconductivity* (Heidelberg: Springer)

[4] Anderson P W 1988 *Frontiers and Borderlines in Many Particle Physics, 'Enrico Fermi' International School of Physics, Course CIV* ed R A Broglia and J R Schrieffer (Amsterdam: North-Holland)
[5] Varma C M, Littlewood P B, Schmitt-Rink S, Abrahams E and Ruckenstein A E 1988 *Phys. Rev. Lett.* **63** 1996
[6] Virosztek A and Ruvalds J 1990 *Phys. Rev.* B **42** 4064
[7] See e.g.,
Pines D and Nozières P 1966 *The Theory of Quantum Liquids* (New York: Benjamin)
[8] Gorkov L P and Dzyaloshinski I E 1973 *Pis. Zh. Eksp. Teor. Fiz.* **18** 686 (Engl. Transl. *Sov. Phys.–JETP Lett.* **18** 401)
[9] Chaplik A V 1971 *Zh. Eksp. Teor. Fiz.* **60** 1845 (Engl. Transl. *Sov. Phys.–JETP* **33** 997)
[10] Hodges C, Smith H and Wilkins J 1971 *Phys. Rev.* B **4** 302
[11] Bloom P 1975 *Phys. Rev.* B **12** 125
[12] Cui S M and Cai J H 1990 *Acta Phys. Sinica* **39** 565
[13] Engelbrecht J R and Randeria M 1990 *Phys. Rev. Lett.* **65** 1032
[14] Lee P A and Read N 1987 *Phys. Rev. Lett.* **58** 2691
[15] See, e.g.,
Abrikosov A A, Gorkov L P and Dzyaloshinski I E 1964 *Methods of Quantum Field Theory in Statistical Physics* (Englewood Cliffs, NJ: Prentice-Hall)
[16] Yamada K and Yosida K 1986 *Prog. Theor. Phys.* **76** 621